

# New smooth hybrid inflation

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We consider the extension of the supersymmetric Pati-Salam model which solves the  $b$ -quark mass problem of supersymmetric grand unified models with exact Yukawa unification and universal boundary conditions and leads to the so-called new shifted hybrid inflationary scenario. We show that this model can also lead to a new version of smooth hybrid inflation based only on renormalizable interactions provided that a particular parameter of its superpotential is somewhat small. The potential possesses valleys of minima with classical inclination, which can be used as inflationary paths. The model is consistent with the fitting of the three-year Wilkinson microwave anisotropy probe data by the standard power-law cosmological model with cold dark matter and a cosmological constant. In particular, the spectral index turns out to be adequately small so that it is compatible with the data. Moreover, the Pati-Salam gauge group is broken to the standard model gauge group during inflation and, thus, no monopoles are formed at the end of inflation. Supergravity corrections based on a non-minimal Kähler potential with a convenient choice of a sign keep the spectral index comfortably within the allowed range without generating maxima and minima of the potential on the inflationary path. So, unnatural restrictions on the initial conditions for inflation can be avoided.

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## I. INTRODUCTION

In recent years, a plethora of precise cosmological observations on the cosmic microwave background radiation and the large-scale structure in the universe has strongly favored the idea of inflation [1] (for a review see e.g. Ref. [2]). Therefore, the construction of realistic models of inflation which are based on particle theory and are consistent with all cosmological and phenomenological requirements is an important task. One of the most promising inflationary models is, undoubtedly, the well-known hybrid inflation [3]. This scenario is [4, 5] naturally realized in the context of supersymmetric (SUSY) grand unified theory (GUT) models based on gauge groups with rank greater than or equal to five.

An attractive rank five gauge group is certainly the Pati-Salam (PS) group  $G_{\text{PS}} = \text{SU}(4)_c \times \text{SU}(2)_L \times \text{SU}(2)_R$  [6]. This is the simplest GUT gauge group which can lead [7] to “asymptotic” Yukawa unification [8], i.e. the exact equality of the third generation Yukawa coupling constants at the GUT scale. Moreover, SUSY PS GUT models are motivated [9] (see also Ref. [10]) from the recent D-brane set-ups and can also arise [11] from the standard weakly coupled heterotic string.

The standard realization of the SUSY hybrid inflation scenario is based on a renormalizable superpotential. In this model, the spontaneous breaking of the GUT gauge symmetry takes place at the end of inflation and, thus, topological defects are copiously formed [12] if they are predicted by this symmetry breaking. The spontaneous breaking of  $G_{\text{PS}}$  to the standard model (SM)

gauge group  $G_{\text{SM}}$  does predict topologically stable magnetic monopoles, which carry [13] two units of Dirac magnetic charge. So, these monopoles are overproduced [14] at the end of standard SUSY hybrid inflation leading to a cosmological disaster.

A possible solution to this problem may be obtained [12, 14, 15] by including into the standard superpotential for hybrid inflation the leading non-renormalizable term, which cannot be excluded by any symmetry and can be comparable with the trilinear term of the standard superpotential. Actually, we have two options. We can either keep [14] both these terms or remove [12] the trilinear term by imposing an appropriate discrete symmetry and keep only the leading non-renormalizable term. In the former case, there appears a new “shifted” classically flat valley of local minima. This valley acquires a slope at the one-loop level and can be used as an alternative inflationary path. The resulting scenario is known as shifted hybrid inflation [14]. The latter option leads to the existence of an inflationary path which possesses an inclination already at the classical level. In contrast to the standard and shifted hybrid inflation scenarios where inflation terminates abruptly and is followed by a “waterfall” regime, in this case, it ends smoothly by saturating the slow-roll conditions. So, the name smooth hybrid inflation was coined [12] for this scenario. In both shifted and smooth hybrid inflation, the GUT gauge group  $G_{\text{PS}}$  is broken to  $G_{\text{SM}}$  already during inflation and thus no topological defects can form at the end of inflation. Consequently, the monopole problem is solved.

It has been shown [16] that shifted hybrid inflation can be realized within the SUSY PS model even without invoking any non-renormalizable superpotential terms provided that we supplement the model with some extra Higgs superfields. This extension of the SUSY PS model was actually introduced [17] (see also Ref. [18]) for a very

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different reason. It is well known [19] that, in SUSY models with exact Yukawa unification (or with large  $\tan\beta$  in general), such as the simplest SUSY PS model, and universal boundary conditions, the  $b$ -quark mass  $m_b$  receives large SUSY corrections, which, for  $\mu > 0$ , lead to unacceptably large values of  $m_b$ . Therefore, Yukawa unification must be (moderately) violated so that, for  $\mu > 0$ , the predicted bottom quark mass resides within the experimentally allowed range even with universal boundary conditions. This requirement forces us to extend the superfield content of this model by including, among other superfields, an extra pair of  $SU(4)_c$  non-singlet  $SU(2)_L$  doublets, which naturally develop [20] subdominant vacuum expectation values (VEVs) and mix with the main electroweak doublets of the model leading to a moderate violation of Yukawa unification. (Note, in passing, that this mechanism applied to the  $\mu < 0$  case, where Yukawa unification predicts a  $m_b$  which after SUSY corrections becomes unacceptably low, does not lead [21] to a viable scheme.) It is remarkable that the resulting extended SUSY PS model automatically and naturally leads [16] to a new version of shifted hybrid inflation based solely on renormalizable superpotential terms. This inflationary scenario was called new shifted hybrid inflation.

In this paper, we show that the same extension of the SUSY PS model can lead to a new version of smooth hybrid inflation based only on renormalizable superpotential terms provided that a particular parameter of its superpotential is adequately small. Indeed, the scalar potential of the model, for a wide range of its other parameters, possesses a valley of minima which has an inclination already at the classical level and can be used as inflationary path leading to a novel realization of smooth hybrid inflation. This scenario will be referred to as new smooth hybrid inflation. The predictions of this inflationary model can be easily made compatible with the recent three-year Wilkinson microwave anisotropy probe (WMAP) measurements [22] for natural values of the parameters of the model. In particular, in global SUSY, the spectral index turns out to be adequately small so that it is consistent with the fitting of the WMAP data [22] by the standard power-law cosmological model with cold dark matter and a cosmological constant ( $\Lambda$ CDM). Finally, as in the “conventional” realization of smooth hybrid inflation,  $G_{PS}$  is already broken to  $G_{SM}$  during new smooth hybrid inflation and, thus, no topological defects are formed at the end of inflation.

The inclusion of supergravity (SUGRA) corrections with minimal Kähler potential raises the spectral index above the allowed range as in standard and shifted hybrid inflation for relatively large values of the relevant dimensionless coupling constant and in smooth hybrid inflation for GUT breaking scale close to its SUSY value (see Ref. [23]). However, the introduction of a non-minimal term in the Kähler potential with appropriately chosen sign can help to reduce the spectral index so that it becomes comfortably compatible with the data (compare with Refs. [24, 25, 26]). This can be achieved with the

potential remaining a monotonically increasing function of the inflaton field everywhere on the inflationary path. So, complications [25, 26] from the appearance of a local maximum and minimum of the potential on the inflationary path when such a non-minimal Kähler potential is used are avoided. One possible complication is that the system gets trapped near the minimum of the inflationary potential and, consequently, no hybrid inflation takes place. Another complication is that, even if hybrid inflation of the so-called hilltop type [27] occurs with the inflaton rolling from the region of the maximum down to smaller values, the spectral index can become compatible with the data only at the cost of a mild tuning of the initial conditions (see Ref. [28]).

The paper is organized as follows. In Sec. II, we briefly introduce the extended SUSY PS model and show that it possesses a valley of minima along which successful new smooth hybrid inflation can take place. In Sec. III, we discuss how our new smooth inflationary scenario is affected by the SUGRA corrections to the scalar potential. Finally, in Sec. IV, we summarize our conclusions.

## II. NEW SMOOTH HYBRID INFLATION IN GLOBAL SUPERSYMMETRY

We consider the extended SUSY PS model of Ref. [17] as our starting point. This model allows a moderate violation of the asymptotic Yukawa unification so that, for  $\mu > 0$ , an acceptable value of the  $b$ -quark mass is obtained even with universal boundary conditions. The breaking of  $G_{PS}$  to  $G_{SM}$  is achieved by the superheavy VEVs ( $= M_{GUT} \simeq 2.86 \cdot 10^{16}$  GeV, the SUSY GUT scale) of the right handed neutrino type components of a conjugate pair of Higgs superfields  $H^c$  and  $\bar{H}^c$  belonging to the  $(4, 1, 2)$  and  $(\bar{4}, 1, 2)$  representations of  $G_{PS}$  respectively. The model also contains a gauge singlet  $S$  and a conjugate pair of superfields  $\phi, \bar{\phi}$  belonging to the  $(15, 1, 3)$  representation of  $G_{PS}$ . The superfield  $\phi$  acquires a (subdominant) VEV which breaks  $G_{PS}$  to  $G_{SM} \times U(1)_{B-L}$ . For details on the full field content and superpotential, the global symmetries, the charge assignments, and the phenomenological and cosmological properties of this model, the reader is referred to Refs. [14, 17] (see also Ref. [18]).

As already mentioned, this extended SUSY PS model leads [16] to a new version of shifted hybrid inflation which is based solely on renormalizable interactions. The superpotential terms which are relevant for this inflationary scenario have been given in Eq. (2.1) of Ref. [16] and have been used there with a particular choice of the phases of their coupling constants. These terms with a different (more convenient for our purposes here) choice of basic parameters and their phases can be written as

$$W = \kappa S(M^2 - \phi^2) - \gamma S H^c \bar{H}^c + m \phi \bar{\phi} - \lambda \bar{\phi} H^c \bar{H}^c, \quad (1)$$

where  $M, m > 0$  are superheavy masses of the order of  $M_{GUT}$  and  $\kappa, \gamma, \lambda > 0$  are dimensionless coupling constants. These parameters are normalized so that they

correspond to the couplings between the SM singlet components of the superfields. In a general superpotential of the type in Eq. (1),  $M$ ,  $m$  and any two of the three dimensionless parameters  $\kappa$ ,  $\gamma$ ,  $\lambda$  can always be made real and positive by appropriately redefining the phases of the superfields. The third dimensionless parameter, however, remains generally complex. For definiteness, we have chosen here this parameter to be real and positive too. One can show that the superpotential in Eq. (2.1) of Ref. [16] with the particular choice of the phases of its parameters considered there can become equivalent to the superpotential in Eq. (1) provided that its real and positive parameter  $\lambda$  is rotated to the negative imaginary axis. Actually, the form of the superpotential in Eq. (1) can be derived from the one in Eq. (2.1) of Ref. [16] by the replacement:  $S \rightarrow -S$ ,  $\phi \rightarrow i\phi$ ,  $\bar{\phi} \rightarrow -i\bar{\phi}$ ,  $\kappa \rightarrow \gamma$ ,  $\beta \rightarrow \kappa$ ,  $\lambda \rightarrow -i\lambda$ ,  $M^2 \rightarrow (\kappa/\gamma)M^2$ .

In this paper, we will show that the specific superpotential of Eq. (1) leads to a new version of smooth hybrid inflation [12] provided that the parameter  $\gamma$  is taken to be adequately small. To this end, we will first examine the case with  $\gamma$  set to zero and then we will move on to allow a small, but non-zero value for this parameter. Note that one could get rid of the  $\gamma$ -term in the superpotential entirely by introducing an extra  $Z_2$  symmetry under which  $H^c$ ,  $\phi$ , and  $\bar{\phi}$  change sign. However, this would disallow the solution of the  $b$ -quark mass problem [17] and, thus, invalidate the original motivation for introducing this extended SUSY PS model. This is due to the fact that the superpotential term which generates the crucial mixing between the  $SU(4)_c$  singlet and non-singlet  $SU(2)_L$  doublets (see Ref. [17]) is forbidden by this discrete symmetry. Needless to say that, for  $\gamma = 0$ , all the choices for the phases of the parameters in Eq. (1) are equivalent.

### A. The $\gamma = 0$ case

Setting  $\gamma = 0$ , the F-term scalar potential obtained from  $W$  is given by

$$V = \kappa^2 |M^2 - \phi^2|^2 + |m\bar{\phi} - 2\kappa S\phi|^2 + |m\phi - \lambda H^c \bar{H}^c|^2 + \lambda^2 |\bar{\phi}|^2 (|H^c|^2 + |\bar{H}^c|^2), \quad (2)$$

where the complex scalar fields which belong to the SM singlet components of the superfields are denoted by the same symbol. We will ignore throughout the soft SUSY breaking terms [29] in the scalar potential since their effect on inflationary dynamics is negligible in our case as in the case of the conventional realization of smooth hybrid inflation (see Ref. [26]).

From the potential in Eq. (2), we find that the SUSY vacua lie at

$$\bar{\phi} = S = 0, \quad \phi^2 = M^2, \quad H^c \bar{H}^c = \frac{m}{\lambda} \phi. \quad (3)$$

The vanishing of the D-terms yields  $\bar{H}^{c*} = e^{i\theta} H^c$ , which

implies that we have four distinct SUSY vacua:

$$\phi = M, \quad H^c = \bar{H}^c = \pm \sqrt{\frac{mM}{\lambda}} \quad (\theta = 0), \quad (4)$$

$$\phi = -M, \quad H^c = -\bar{H}^c = \pm \sqrt{\frac{mM}{\lambda}} \quad (\theta = \pi) \quad (5)$$

with  $\bar{\phi} = S = 0$ . Here, for simplicity,  $H^c$ ,  $\bar{H}^c$  have been rotated to the real axis by an appropriate gauge transformation. However, we should keep in mind that the fields  $H^c$ ,  $\pm \bar{H}^{c*}$  (the plus or minus sign corresponds to  $\theta = 0$  or  $\pi$  respectively) can have an arbitrary common phase in the vacuum and, thus, the two distinct vacua in Eq. (4) or (5) are not, in reality, discrete, but rather belong to a continuous  $S^1$  vacuum submanifold. Note that the vacua in Eq. (4) are related to the ones in Eq. (5) by the  $Z_2$  symmetry mentioned above. As we will see later, the specific point of the vacuum manifold towards which the system is heading is already chosen during inflation. So the model does not encounter any topological defect problem. Actually, there is no production of topological defects at all.

It is not very hard to show that, at any possible minimum of the potential,  $\epsilon = 0$  or  $\pi$  and  $\epsilon = \bar{\epsilon} = -\theta$ , where  $\epsilon$  and  $\bar{\epsilon}$  are the phases of  $\phi$  and  $\bar{\phi}$  respectively ( $S$  can be made real by an appropriate global  $U(1)$  R transformation). This remains true even at the minima of  $V$  with respect to  $\phi$ ,  $\bar{\phi}$ ,  $H^c$ , and  $\bar{H}^c$  for fixed  $S$ . So, we will restrict ourselves to these values of  $\theta$  and the phases of  $\phi$  and  $\bar{\phi}$ . The scalar potential then takes the form

$$V_{\min} = \kappa^2 (|\phi|^2 - M^2)^2 + (2\kappa|S||\phi| - m|\bar{\phi}|)^2 + (m|\phi| - \lambda|H^c|^2)^2 + 2\lambda^2|\bar{\phi}|^2|H^c|^2. \quad (6)$$

The derivatives of this potential with respect to the norms of the fields are

$$\frac{\partial V_{\min}}{\partial |S|} = 4\kappa (2\kappa|S||\phi| - m|\bar{\phi}|) |\phi|, \quad (7)$$

$$\frac{\partial V_{\min}}{\partial |\phi|} = 4\kappa^2 (|\phi|^2 - M^2) |\phi| + 4\kappa (2\kappa|S||\phi| - m|\bar{\phi}|) |S| + 2m (m|\phi| - \lambda|H^c|^2), \quad (8)$$

$$\frac{\partial V_{\min}}{\partial |\bar{\phi}|} = -2m (2\kappa|S||\phi| - m|\bar{\phi}|) + 4\lambda^2 |\bar{\phi}| |H^c|^2, \quad (9)$$

$$\frac{\partial V_{\min}}{\partial |H^c|} = -4\lambda (m|\phi| - \lambda|H^c|^2 - \lambda|\bar{\phi}|^2) |H^c|. \quad (10)$$

The potential  $V_{\min}$  possesses two flat directions. The first one is the trivial flat direction at  $|\phi| = |\bar{\phi}| = |H^c| = 0$  with  $V = V_{\text{tr}} \equiv \kappa^2 M^4$ . The second one exists only if  $\tilde{\mu}^2 \equiv M^2 - m^2/2\kappa^2 > 0$  and is a shifted flat direction at

$$|\phi| = \tilde{\mu}, \quad |\bar{\phi}| = \frac{2\kappa\tilde{\mu}}{m} |S|, \quad |H^c| = 0, \quad (11)$$

where  $\tilde{\mu} \equiv (M^2 - m^2/2\kappa^2)^{1/2}$ , with  $V = \kappa^2 (M^4 - \tilde{\mu}^4)$ . The mass-squared matrix of the variables  $|S|$ ,  $|\phi|$ ,  $|\bar{\phi}|$ ,

and  $|H^c|$  on the trivial flat direction is

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 4\kappa^2(2|S|^2 - \tilde{\mu}^2) & -4\kappa m|S| & 0 \\ 0 & -4\kappa m|S| & 2m^2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \quad (12)$$

If  $M_{\phi\bar{\phi}}$  denotes the  $|\phi|, |\bar{\phi}|$  sector of this matrix, then

$$\det(M_{\phi\bar{\phi}}) = -8\kappa^2 m^2 \tilde{\mu}^2, \quad (13)$$

$$\text{tr}(M_{\phi\bar{\phi}}) = 4\kappa^2(2|S|^2 - \tilde{\mu}^2) + 2m^2. \quad (14)$$

So, the matrix  $M_{\phi\bar{\phi}}$  has one positive and one negative eigenvalue for  $\tilde{\mu}^2 > 0$  and two positive eigenvalues for  $\tilde{\mu}^2 < 0$ . In the former case, the trivial flat direction is a path of saddle points and the shifted flat direction is an honest candidate for the inflationary path. However, in this paper, we will concentrate on the latter case and set  $\mu^2 \equiv -\tilde{\mu}^2 > 0$ . Note that, even in this case, the trivial flat direction may not be a valley of local minima because of the existence of the zero eigenvalue of the full mass-squared matrix in Eq. (12) associated with the field  $|H^c|$ . It is perfectly conceivable that, starting from any point on the trivial flat direction, there exist paths along which the potential decreases as we move away from this flat direction (at least initially). Actually, as we will show below, this happens to be the case here.

To examine the stability of the trivial flat direction, we consider a point on it and try to see whether, starting from this point, one can construct paths in the  $(|H^c|, |\phi|, |\bar{\phi}|)$  space along which the potential in Eq. (6) has a local maximum at the point on the trivial flat direction. In particular, we will try to find the path of steepest descent. Throughout the analysis,  $|S|$  will be considered as a fixed parameter characterizing the chosen point on the trivial flat direction rather than as a dynamical variable. Setting  $|H^c| = \chi$ ,  $|\phi| = \psi$ , and  $|\bar{\phi}| = \omega$ , we can parameterize any path in the field space as  $(\chi, \psi(\chi), \omega(\chi))$ . We see, from the form of the matrix in Eq. (12), that the required paths must be tangential to the  $|H^c|$  direction at their origin (because, for  $\mu^2 > 0$ , displacement along the  $|\phi|$  or  $|\bar{\phi}|$  direction enhances the potential locally). Thus, the required initial conditions for these paths are

$$\chi = 0, \quad \psi(0) = \omega(0) = 0, \quad \psi'(0) = \omega'(0) = 0, \quad (15)$$

where prime denotes derivation with respect to  $\chi$ .

The potential  $V_{\min}$  on such a path can be written as

$$F(\chi) = f(\chi, \psi(\chi), \omega(\chi)), \quad (16)$$

where  $f(\chi, \psi, \omega) \equiv V_{\min}(\chi, \psi, \omega)$ . It is then obvious that  $F'(0)$  is zero by construction since

$$(\bar{\nabla} V_{\min})_0 = 0, \quad (17)$$

where  $(\dots)_0$  denotes the value at  $\chi = \psi = \omega = 0$ . Thus, the initial point of the path is a critical point of  $F(\chi)$  (as it should). Moreover, it is easily verified, using Eqs. (12),

(15), and (17), that  $F''(0) = 0$ , which means that we cannot decide on the stability of the trivial flat direction merely from the mass-squared matrix in Eq. (12). Therefore, higher derivatives of  $F(\chi)$  must be considered. We find that  $F'''(0) = 0$  and

$$F''''(0) = \alpha + \zeta \psi_0'' + \rho \omega_0'' + (\psi_0'', \omega_0'') \begin{pmatrix} a & c \\ c & b \end{pmatrix} \begin{pmatrix} \psi_0'' \\ \omega_0'' \end{pmatrix} \quad (18)$$

with  $\psi_0'' \equiv \psi''(0)$ ,  $\omega_0'' \equiv \omega''(0)$ ,

$$\begin{aligned} \alpha &\equiv \left( \frac{\partial^4 f}{\partial \chi^4} \right)_0 = 24\lambda^2, \\ \zeta &\equiv 6 \left( \frac{\partial^3 f}{\partial \chi^2 \partial \psi} \right)_0 = -24\lambda m, \\ \rho &\equiv 6 \left( \frac{\partial^3 f}{\partial \chi^2 \partial \omega} \right)_0 = 0, \\ a &\equiv 3 \left( \frac{\partial^2 f}{\partial \psi^2} \right)_0 = 12\kappa^2(\mu^2 + 2|S|^2), \\ b &\equiv 3 \left( \frac{\partial^2 f}{\partial \omega^2} \right)_0 = 6m^2, \\ c &\equiv 3 \left( \frac{\partial^2 f}{\partial \psi \partial \omega} \right)_0 = -12\kappa m|S|, \end{aligned}$$

where Eqs. (6), (15), and (17) were used. Note that the  $2 \times 2$  matrix in the last term in the right hand side of Eq. (18) is just  $3M_{\phi\bar{\phi}}$ , which is positive definite for  $\mu^2 > 0$  (see the discussion following Eq. (12)).

By applying the transformation

$$\psi_0'' = \hat{\psi}_0'' + \delta\psi_0'', \quad \omega_0'' = \hat{\omega}_0'' + \delta\omega_0'', \quad (19)$$

one can show that Eq. (18) can be brought into the form

$$F''''(0) = -\frac{24\lambda^2 M^2}{\mu^2} + (\delta\psi_0'', \delta\omega_0'') \begin{pmatrix} a & c \\ c & b \end{pmatrix} \begin{pmatrix} \delta\psi_0'' \\ \delta\omega_0'' \end{pmatrix} \quad (20)$$

with

$$\hat{\psi}_0'' = -\frac{\zeta b}{2(ab - c^2)} > 0, \quad \hat{\omega}_0'' = \frac{\zeta c}{2(ab - c^2)} \geq 0. \quad (21)$$

The last term in the right hand side of Eq. (20) is a positive definite quadratic form in  $\delta\psi_0'' \geq -\hat{\psi}_0''$ ,  $\delta\omega_0'' \geq -\hat{\omega}_0''$  (the non-positive lower bounds originate from the fact that  $\psi_0'', \omega_0'' \geq 0$ , which in turn comes from Eq. (15) and the fact that  $\psi, \omega \geq 0$  by their definition). It is obvious then that there exist choices of  $\delta\psi_0'', \delta\omega_0''$  which render  $F_0''''$  negative. Thus, on the corresponding paths,  $F(\chi)$  has a local maximum at  $\chi = 0$ . We conclude that the trivial flat direction is a path of saddle points rather than a valley of local minima. The path of steepest descent corresponds to  $\delta\psi_0'' = \delta\omega_0'' = 0$ , which minimizes  $F_0''''$ .

We have just seen that, for any fixed value of  $|S|$ ,  $V_{\min}$  has a local maximum on the trivial flat direction at  $|\phi| = |\bar{\phi}| = |H^c| = 0$ . Moreover,  $V_{\min} \rightarrow +\infty$  as  $|\phi|^2 + |\bar{\phi}|^2 + |H^c|^2 \rightarrow \infty$ . This means that, for each value

of  $|S|$ ,  $V_{\min}$  must have a non-trivial absolute minimum (where at least one the fields  $|\phi|$ ,  $|\bar{\phi}|$ , and  $|H^c|$  has a non-zero value). These minima then form a valley, which may be used as inflationary trajectory. Actually, as we will show soon, this trajectory is not flat and resembles the path used in Ref. [12] for smooth hybrid inflation. We can find the valley of minima of  $V_{\min}$  by minimizing this potential with respect to  $|\phi|$ ,  $|\bar{\phi}|$ , and  $|H^c|$ , regarding  $|S|$  as a fixed parameter. This amounts to solving the system of equations that is formed by equating the partial derivatives in Eqs. (8)-(10) with zero. We obtain three non-linear equations with three unknowns, which cannot be solved analytically. Though, as in the case of conventional smooth hybrid inflation [12], we will try to find a solution in the large  $|S|$  limit. In particular, we will try to find a power series solution with respect to some parameter of the form “mass”/ $|S|$  which remains smaller than unity throughout the entire range of  $|S|$  which is relevant for inflation. As it will become clear below, a convenient quantity for the “mass” in the numerator is  $v_g \equiv \sqrt{mM/\lambda}$ , which is just the VEV  $|\langle H^c \rangle|$  at the SUSY minima of the potential. Re-expressing the system of equations by using the dimensionless variables  $x \equiv |\phi|/M$ ,  $y \equiv |\bar{\phi}|/\sqrt{2}p v_g$ ,  $z \equiv |H^c|/v_g$ , and  $w \equiv v_g/|S|$ , where  $p \equiv \sqrt{2}\kappa M/m$  is a dimensionless parameter smaller than unity for  $\mu^2 > 0$ , we obtain

$$\begin{aligned} wx(x^2 - 1) + 4yz^2 + 2wy^2 &= 0, \\ x - wy &= \sqrt{2} \frac{\lambda}{\kappa} pwyz^2, \\ x &= z^2 + 2p^2y^2. \end{aligned} \quad (22)$$

Writing the variables  $x$ ,  $y$ , and  $z^2$  as power series in  $w$  and equating the coefficients of the corresponding powers of  $w$  in the two sides of Eqs. (22), we get

$$\begin{aligned} x &= x_2w^2 + x_4w^4 + \dots, \\ y &= y_1w + y_3w^3 + \dots, \\ z^2 &= z_2w^2 + z_4w^4 + \dots, \end{aligned} \quad (23)$$

where the coefficients  $x_i$ ,  $y_i$ , and  $z_i$  depend only on the parameter  $p$  and the ratio  $\lambda/\kappa$  and are given by

$$x_2 = y_1 = \frac{3}{8p^2} \left(1 - \sqrt{1 - 8p^2/9}\right), \quad (24)$$

$$z_2 = \frac{1}{4} (1 - 2x_2), \quad (25)$$

$$x_4 = \frac{\sqrt{2}}{8} \frac{\lambda}{\kappa} p \frac{x_2(1 - 2x_2)(3 - 10x_2)}{1 - 3x_2}, \quad (26)$$

$$y_3 = \frac{1 - 4x_2}{3 - 10x_2} x_4, \quad (27)$$

$$z_4 = \frac{1 + 2(1 - 2p^2)x_2}{3 - 10x_2} x_4. \quad (28)$$

A useful approximation to these coefficients can be found by expanding them with respect to the small parameter  $p$  (see below). Thus, to first non-trivial order in

$p$ , we find the following simple expressions:

$$x_2 = y_1 = z_2 = \frac{1}{6}, \quad (29)$$

$$x_4 = z_4 = \frac{\sqrt{2}}{27} \frac{\lambda}{\kappa} p, \quad y_3 = \frac{\sqrt{2}}{108} \frac{\lambda}{\kappa} p \quad (30)$$

and Eq. (23) takes the form

$$\begin{aligned} |\phi| &\simeq \frac{Mv_g^2}{6|S|^2} \left(1 + \frac{2\sqrt{2}}{9} \frac{\lambda}{\kappa} p w^2 + \dots\right), \\ |\bar{\phi}| &\simeq \sqrt{2}p \frac{v_g^2}{6|S|} \left(1 + \frac{\sqrt{2}}{18} \frac{\lambda}{\kappa} p w^2 + \dots\right), \\ |H^c| &\simeq \frac{v_g^2}{\sqrt{6}|S|} \left(1 + \frac{\sqrt{2}}{9} \frac{\lambda}{\kappa} p w^2 + \dots\right). \end{aligned} \quad (31)$$

Taking into account the possible values of the phases  $\epsilon$ ,  $\bar{\epsilon}$ , and  $\theta$  (and with  $H^c$ ,  $\bar{H}^c$  rotated to the real axis), we see that the potential in Eq. (2) possesses four valleys of absolute minima (for fixed  $|S|$ ) which presumably lead to the four SUSY vacua in Eqs. (4) and (5). We should keep in mind, though, that the two valleys corresponding to the same value of  $\theta$  are not discrete, but continuously connected since  $H^c$ ,  $\pm \bar{H}^{c*}$  can have an arbitrary common phase. The expansions in Eq. (31) hold as long as  $w < 1$ , that is  $|S| > v_g$ . In the following, we will keep only the terms of leading order in  $w$  in the above equations. Although this might seem somewhat arbitrary, we will justify it later. Substituting the expansions in Eq. (31) into the potential of Eq. (6) and keeping only terms of leading order in  $w$ , we get

$$V_{\min} \simeq \kappa^2 M^4 \left(1 - \frac{v_g^4}{54|S|^4}\right). \quad (32)$$

This is exactly the form of the potential for smooth hybrid inflation considered in Ref. [12]. Thus, we have shown that the present model possesses inflationary paths leading to smooth hybrid inflation. We will call the resulting scenario new smooth hybrid inflation since, in contrast to the conventional realization of smooth hybrid inflation, it is achieved by using only renormalizable interactions. It is evident that, as the system follows the new smooth inflationary path, the phases of the various fields remain fixed. Moreover, the particular point of the vacuum manifold towards which the system is heading is already chosen during inflation and we encounter no cosmological defect problems.

Setting  $S = \sigma/\sqrt{2}$ , where  $\sigma$  is the canonically normalized real inflaton field (recall that  $S$  was made real by an R transformation), we obtain the potential along the new smooth inflationary path

$$V \simeq v_0^4 \left(1 - \frac{2v_g^4}{27\sigma^4}\right), \quad (33)$$

where  $v_0 \equiv \sqrt{\kappa}M$  is the inflationary scale. The slow-roll parameters  $\varepsilon$ ,  $\eta$  and the parameter  $\xi^2$ , which enters the running of the spectral index, are (see e.g. Ref. [30])

$$\varepsilon \equiv \frac{m_{\text{P}}^2}{2} \left( \frac{V^{(1)}(\sigma)}{V(\sigma)} \right)^2 \simeq \frac{32m_{\text{P}}^2 v_g^8}{729\sigma^{10}}, \quad (34)$$

$$\eta \equiv m_{\text{P}}^2 \left( \frac{V^{(2)}(\sigma)}{V(\sigma)} \right) \simeq -\frac{40m_{\text{P}}^2 v_g^4}{27\sigma^6}, \quad (35)$$

$$\xi^2 \equiv m_{\text{P}}^4 \left( \frac{V^{(1)}(\sigma)V^{(3)}(\sigma)}{V^2(\sigma)} \right) \simeq \frac{640m_{\text{P}}^4 v_g^8}{243\sigma^{12}}, \quad (36)$$

where the superscript  $(n)$  denotes the  $n$ -th derivative with respect to  $\sigma$  and  $m_{\text{P}}$  is the reduced Planck mass. Inflation ends at  $\sigma = \sigma_f$  (taken positive by an R transformation) where  $\eta = -1$ , which gives

$$\sigma_f^6 \simeq \frac{40m_{\text{P}}^2 v_g^4}{27}. \quad (37)$$

The number of e-foldings from the time when the pivot scale  $k_0 = 0.002 \text{ Mpc}^{-1}$  crosses outside the inflationary horizon until the end of inflation is given (see e.g. Ref. [30]) by

$$N_Q \simeq \frac{1}{m_{\text{P}}^2} \int_{\sigma_f}^{\sigma_Q} \frac{V(\sigma)}{V^{(1)}(\sigma)} d\sigma \simeq \frac{9}{16m_{\text{P}}^2 v_g^4} (\sigma_Q^6 - \sigma_f^6), \quad (38)$$

where  $\sigma_Q \equiv \sqrt{2}S_Q > 0$  is the value of the inflaton field at horizon crossing of the pivot scale. Taking into account the fact that  $\sigma_f \ll \sigma_Q$ , we can write

$$\sigma_Q^6 \simeq \frac{16m_{\text{P}}^2 v_g^4}{9} N_Q. \quad (39)$$

The power spectrum  $P_{\mathcal{R}}$  of the primordial curvature perturbation at the scale  $k_0$  is given (see e.g. Ref. [30]) by

$$P_{\mathcal{R}}^{1/2} \simeq \frac{1}{2\pi\sqrt{3}} \frac{V^{3/2}(\sigma_Q)}{m_{\text{P}}^3 V^{(1)}(\sigma_Q)} \simeq \frac{3^{5/6} N_Q^{5/6}}{2^{2/3}\pi} \left( \frac{v_0^3}{m_{\text{P}}^2 v_g} \right)^{2/3}. \quad (40)$$

The spectral index  $n_s$ , the tensor-to-scalar ratio  $r$ , and the running of the spectral index  $dn_s/d \ln k$  are given (see e.g. Ref. [30]) by

$$\begin{aligned} n_s &\simeq 1 + 2\eta - 6\varepsilon \simeq 1 - \frac{5}{3N_Q}, \\ r &\simeq 16\varepsilon \simeq \frac{2^{7/3}}{3^{8/3} N_Q^{5/3}} \left( \frac{v_g}{m_{\text{P}}} \right)^{4/3}, \\ \frac{dn_s}{d \ln k} &\simeq 16\varepsilon\eta - 24\varepsilon^2 - 2\xi^2 \simeq -\frac{5}{3N_Q^2}, \end{aligned} \quad (41)$$

where  $\varepsilon$ ,  $\eta$ , and  $\xi^2$  are evaluated at  $\sigma = \sigma_Q$ . The number of e-foldings  $N_Q$  required for solving the horizon and flatness problems of standard hot big bang cosmology is given (see e.g. Ref. [2]) approximately by

$$N_Q \simeq 53.76 + \frac{2}{3} \ln \left( \frac{v_0}{10^{15} \text{ GeV}} \right) + \frac{1}{3} \ln \left( \frac{T_r}{10^9 \text{ GeV}} \right), \quad (42)$$

where  $T_r$  is the reheat temperature which is expected not to exceed about  $10^9 \text{ GeV}$ , which is the well-known gravitino bound [31].

Taking  $v_g$  to have the SUSY GUT value, i.e.  $v_g \simeq 2.86 \cdot 10^{16} \text{ GeV}$  (see below),  $T_r$  to saturate the gravitino bound, i.e.  $T_r \simeq 10^9 \text{ GeV}$ , and the WMAP [22] normalization  $P_{\mathcal{R}}^{1/2} \simeq 4.85 \cdot 10^{-5}$  at the comoving scale  $k_0$ , we can solve Eqs. (40) and (42) numerically. We obtain

$$N_Q \simeq 53.78, \quad v_0 \simeq 1.036 \cdot 10^{15} \text{ GeV}. \quad (43)$$

The spectral index, the tensor-to-scalar ratio, and the running of the spectral index are then

$$n_s \simeq 0.969, \quad r \simeq 9.4 \cdot 10^{-7}, \quad \frac{dn_s}{d \ln k} \simeq -5.8 \cdot 10^{-4}. \quad (44)$$

We see that the running of the spectral index and the tensor-to-scalar ratio are negligible and, thus, the standard power-law  $\Lambda\text{CDM}$  cosmological model should hold to a very good accuracy. Fitting the three-year results from WMAP [22] with this cosmological model, one obtains that, at the pivot scale  $k_0$ ,

$$n_s = 0.958 \pm 0.016 \Rightarrow 0.926 \lesssim n_s \lesssim 0.99 \quad (45)$$

at 95% confidence level. So, the value of the spectral index in Eq. (44) is perfectly acceptable. It is, actually, the same as in conventional smooth hybrid inflation [12] since the inflationary potential for large  $|S|$  is exactly the same, as we already pointed out.

We have already fixed the values of the parameters  $v_0 = \sqrt{\kappa}M$  and  $v_g = \sqrt{mM/\lambda}$ . So, we are free to make two more choices in order to determine the four parameters of the model  $m$ ,  $M$ ,  $\kappa$ , and  $\lambda$ . A legitimate choice is to set  $\kappa = \lambda$  and  $m = M$  which leads to quite natural values for the parameters, namely

$$\begin{aligned} m = M &= \sqrt{v_0 v_g} \simeq 5.44 \cdot 10^{15} \text{ GeV}, \\ \kappa = \lambda &= \frac{v_0}{v_g} \simeq 0.0362. \end{aligned} \quad (46)$$

For these values, we find, from Eqs. (37) and (39), that  $\sigma_f \simeq 1.34 \cdot 10^{17} \text{ GeV}$  and  $\sigma_Q \simeq 2.69 \cdot 10^{17} \text{ GeV}$ .

Let us now turn to the justification of the expansions in Eqs. (23) and (31). The value of  $|S|$  at the termination of inflation is approximately given by

$$S_f^6 = \frac{\sigma_f^6}{2^3} \simeq \frac{5m_{\text{P}}^2 v_g^4}{27}. \quad (47)$$

Therefore, the maximum value of  $w$  during inflation is

$$w_{\text{max}} = \frac{v_g}{S_f} \simeq \frac{3^{1/2}}{5^{1/6}} \left( \frac{v_g}{m_{\text{P}}} \right)^{1/3} \simeq 0.3. \quad (48)$$

Consequently, the condition  $w < 1$  is well satisfied during inflation and the expansions in Eq. (23) are valid. Moreover,  $p \simeq 0.0512 \ll 1$  for the values in Eq. (46) and, thus, for  $\lambda \sim \kappa$ , the expansions in Eq. (31) are also justified.

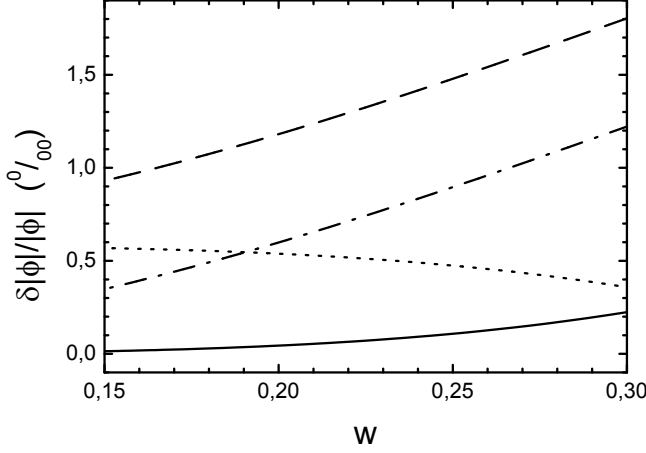


FIG. 1: Relative error in  $|\phi|$  on the new smooth inflationary path in global SUSY for the values of the parameters in Eq. (46) and  $\gamma = 0$  when we use the expansion of Eq. (23) up to second order in  $w$  with coefficient evaluated to leading order in  $p$  (dashed line) or accurately (dot-dashed line) and up to fourth order in  $w$  with coefficients evaluated to leading order in  $p$  (dotted line) or accurately (solid line).

We find numerically that these expansions are, actually, justified in the entire range  $w \leq w_{\max}$  even for values of  $p$  close to unity and  $\lambda > \kappa$ . Rough estimates of the maximum relative errors when only the leading order term is kept in the expansions of Eq. (31) are given by the second term in the parentheses in this equation for  $w = w_{\max}$ . For the values in Eq. (46), we get that the maximum relative error in  $|\phi|$ , which seems to be the largest of the errors in  $|\phi|$ ,  $|\bar{\phi}|$ , and  $|H^c|$ , is given by the estimate

$$\frac{\delta|\phi|}{|\phi|} \simeq \frac{2\sqrt{2}}{9} \frac{\lambda}{\kappa} p w_{\max}^2 \simeq 1.45 \cdot 10^{-3} \sim 1\%. \quad (49)$$

This is verified numerically as shown in Fig. 1, where we plot the relative error in  $|\phi|$  during inflation when we approximate the new smooth inflationary path by the expansions in Eq. (23). Note that, in order to retain a precision better than 1% in  $|\phi|$  keeping only the leading order term in its expansion in Eq. (31), the relation  $M \lesssim v_g/2$  has to hold, as can be seen from Eq. (49) for  $w_{\max} \simeq 0.3$ .

The identification of  $v_g$ , which is the VEV  $|\langle H^c \rangle|$  or  $|\langle \bar{H}^c \rangle|$ , with the SUSY GUT scale  $M_{\text{GUT}}$  can be easily justified. As already mentioned, the VEVs of  $H^c$ ,  $\bar{H}^c$  break the PS gauge group to  $G_{\text{SM}}$ , whereas the VEV of the field  $\phi$  breaks it only to  $G_{\text{SM}} \times \text{U}(1)_{B-L}$ . So, the gauge boson  $A^\perp$  corresponding to the linear combination of  $\text{U}(1)_Y$  and  $\text{U}(1)_{B-L}$  which is perpendicular to  $\text{U}(1)_Y$  acquires its mass squared  $m_{A^\perp}^2 = (5/2)g^2|\langle H^c \rangle|^2$  solely from the VEVs of  $H^c$ ,  $\bar{H}^c$  ( $g$  is the SUSY GUT gauge coupling constant). On the other hand, the masses squared  $m_A^2$  and  $m_{W_R}^2$  of the color triplet, anti-triplet ( $A^\pm$ ) and charged  $\text{SU}(2)_R$  ( $W_R^\pm$ ) gauge bosons get contributions from  $\langle \phi \rangle$  too. Namely,  $m_A^2 = g^2(|\langle H^c \rangle|^2 + (4/3)|\langle \phi \rangle|^2)$

and  $m_{W_R}^2 = g^2(|\langle H^c \rangle|^2 + 2|\langle \phi \rangle|^2)$ . For the values in Eq. (46), however,

$$\frac{|\langle \phi \rangle|^2}{|\langle H^c \rangle|^2} = \frac{\lambda M}{m} \simeq 0.0362 \ll 1, \quad (50)$$

which implies that  $m_A \approx m_{W_R} \approx gv_g$  within a few per cent. So,  $v_g$  is approximately equal to the practically common mass of the SM non-singlet superheavy gauge bosons divided by  $g \approx 0.7$ , which is, in turn, equal to  $M_{\text{GUT}} \simeq 2.86 \cdot 10^{16}$  GeV (the SM singlet gauge boson  $A^\perp$  does not affect the renormalization group equations).

## B. The $\gamma \neq 0$ case

We will now turn to the case of a non-vanishing, but small value of the parameter  $\gamma$ . The scalar potential, in this case, takes the form

$$\begin{aligned} V = & |\kappa(M^2 - \phi^2) - \gamma H^c \bar{H}^c|^2 \\ & + |m\bar{\phi} - 2\kappa S\phi|^2 + |m\phi - \lambda H^c \bar{H}^c|^2 \\ & + |\gamma S + \lambda \bar{\phi}|^2 (|H^c|^2 + |\bar{H}^c|^2) \end{aligned} \quad (51)$$

and the SUSY vacua lie at

$$\phi = \frac{\gamma m}{2\kappa\lambda} \left( -1 \pm \sqrt{1 + \frac{4\kappa^2\lambda^2 M^2}{\gamma^2 m^2}} \right) \equiv \phi_\pm, \quad (52)$$

$$\bar{\phi} = S = 0, \quad H^c \bar{H}^c = \frac{m}{\lambda} \phi. \quad (53)$$

Again, the vanishing of the D-terms yields  $\bar{H}^{c*} = e^{i\theta} H^c$ , which implies that we have four distinct SUSY vacua:

$$\phi = \phi_+, \quad H^c = \bar{H}^c = \pm \sqrt{\frac{m\phi_+}{\lambda}} \quad (\theta = 0), \quad (54)$$

$$\phi = \phi_-, \quad H^c = -\bar{H}^c = \pm \sqrt{\frac{-m\phi_-}{\lambda}} \quad (\theta = \pi) \quad (55)$$

with  $\bar{\phi} = S = 0$ . Here  $H^c$ ,  $\bar{H}^c$  are rotated to the real axis, but we should again keep in mind that the two vacua in Eq. (54) or (55) belong, in reality, to a continuum of vacua. One can show that the potential now generally possesses three flat directions. The first one is the usual trivial flat direction at  $\phi = \bar{\phi} = H^c = \bar{H}^c = 0$  with  $V = V_{\text{tr}} = \kappa^2 M^4$ . The second one exists only if  $\tilde{\mu}^2 > 0$  and lies at

$$\phi = \pm \tilde{\mu}, \quad \bar{\phi} = \frac{2\kappa\phi}{m} S, \quad H^c = \bar{H}^c = 0. \quad (56)$$

It is a shifted flat direction with  $V = \kappa^2(M^4 - \tilde{\mu}^4)$  along which  $G_{\text{PS}}$  is broken to  $G_{\text{SM}} \times \text{U}(1)_{B-L}$ . Note that the positions of the trivial and shifted flat directions remain the same as in the  $\gamma = 0$  case. The third flat direction,

which appears at

$$\phi = -\frac{\gamma m}{2\kappa\lambda}, \quad \bar{\phi} = -\frac{\gamma}{\lambda} S, \quad (57)$$

$$H^c \bar{H}^c = \frac{\kappa\gamma(M^2 - \phi^2) + \lambda m\phi}{\gamma^2 + \lambda^2}, \quad (58)$$

$$V = V_{\text{nsh}} \equiv \frac{\kappa^2\lambda^2}{\gamma^2 + \lambda^2} \left( M^2 + \frac{\gamma^2 m^2}{4\kappa^2\lambda^2} \right)^2, \quad (59)$$

exists only for  $\gamma \neq 0$  and is analogous to the trajectory for the new shifted hybrid inflation of Ref. [16]. Along this direction,  $G_{\text{PS}}$  is broken to  $G_{\text{SM}}$ . In our subsequent discussion, we will again concentrate on the case where  $\mu^2 = -\tilde{\mu}^2 > 0$ . It is interesting to note that, in this case, we always have  $V_{\text{nsh}} > V_{\text{tr}}$  and it is, thus, more likely that the system will eventually settle down on the trivial rather than the new shifted flat direction (the shifted flat direction in Eq. (56) does not exist in this case).

If we expand the complex scalar fields  $\phi$ ,  $\bar{\phi}$ ,  $H^c$ ,  $\bar{H}^c$  in real and imaginary parts according to the prescription  $s = (s_1 + i s_2)/\sqrt{2}$ , we find that, on the trivial flat direction, the mass-squared matrices  $M_{\phi_1}^2$  of  $\phi_1$ ,  $\bar{\phi}_1$  and  $M_{\phi_2}^2$  of  $\phi_2$ ,  $\bar{\phi}_2$  are

$$M_{\phi_1(\phi_2)}^2 = \begin{pmatrix} m^2 + 4\kappa^2|S|^2 \mp 2\kappa^2 M^2 & -2\kappa m S \\ -2\kappa m S & m^2 \end{pmatrix} \quad (60)$$

and the mass-squared matrices  $M_{H_1}^2$  of  $H_1^c$ ,  $\bar{H}_1^c$  and  $M_{H_2}^2$  of  $H_2^c$ ,  $\bar{H}_2^c$  are

$$M_{H_1(H_2)}^2 = \begin{pmatrix} \gamma^2|S|^2 & \mp\gamma\kappa M^2 \\ \mp\gamma\kappa M^2 & \gamma^2|S|^2 \end{pmatrix}. \quad (61)$$

The matrices  $M_{\phi_1(\phi_2)}^2$  are always positive definite, while the matrices  $M_{H_1(H_2)}^2$  acquire one negative eigenvalue for

$$|S| < S_c \equiv \sqrt{\frac{\kappa}{\gamma}} M. \quad (62)$$

Thus, the trivial flat direction is now stable for  $|S| > S_c$  and unstable for  $|S| < S_c$ . Yet, one can easily see that, for  $\gamma \rightarrow 0$ ,  $S_c \rightarrow \infty$  and we are led to the previous ( $\gamma = 0$ ) case where the entire trivial flat direction was a path of saddle points. So, one can imagine that, for small enough values of the parameter  $\gamma$ , the trivial flat direction, after its destabilization at the critical point, forks into four valleys of local or global minima (for fixed  $|S|$ ) of the potential in Eq. (51), which resemble the valleys for new smooth hybrid inflation described above in the  $\gamma = 0$  case.

Actually, the valleys for a small non-zero  $\gamma$  are expected to differ from the ones for  $\gamma = 0$  by corrections involving the small parameter  $\gamma$ . The terms in the potential of Eq. (51) which depend on  $\gamma$  and the phases  $\epsilon$ ,  $\bar{\epsilon}$ , and  $\theta$  are

$$\begin{aligned} \delta V = & -2\gamma|H^c|^2 \left( \kappa M^2 \cos\theta - 2\lambda|S||\bar{\phi}| \cos\bar{\epsilon} \right. \\ & \left. - \kappa|\phi|^2 \cos(2\epsilon + \theta) \right). \end{aligned} \quad (63)$$

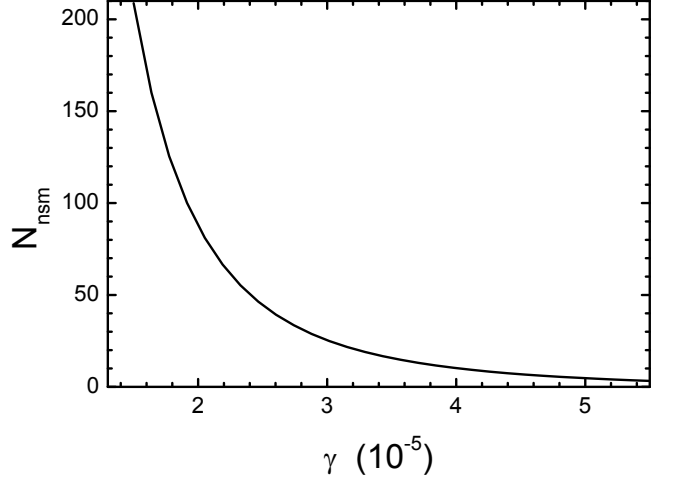


FIG. 2: Number of e-foldings  $N_{\text{nsm}}$  along the new smooth inflationary path versus  $\gamma$  in global SUSY when the system slowly rolls from  $\sigma = 0.95\sigma_c$  down to  $\sigma = \sigma_f$ . The other parameters of the model (except  $\gamma$ ) take the values in Eq. (46).

Estimating this expression on the valleys for  $\gamma = 0$  by using the leading term in the expansion of  $|\phi|$  and  $|\bar{\phi}|$  in Eq. (31), we find that, for  $v_g/|S| < 1$ ,

$$\begin{aligned} \delta V \approx & -2\kappa\gamma M^2 |H^c|^2 \left( \cos\theta - \frac{2}{3} \cos\bar{\epsilon} \right. \\ & \left. - \frac{1}{36} \left( \frac{v_g}{|S|} \right)^4 \cos(2\epsilon + \theta) \right). \end{aligned} \quad (64)$$

From this, we see that the  $\gamma$  dependent corrections enhance the potential in the valleys with  $\epsilon = \bar{\epsilon} = \theta = \pi$  and reduce it in the valleys with  $\epsilon = \bar{\epsilon} = \theta = 0$ . This fact can also be confirmed numerically. So, as it turns out, the trivial flat direction bifurcates at  $|S| = S_c$  into two valleys of *absolute* minima for fixed  $|S|$  which correspond to  $\theta \simeq 0$  and lead to the two SUSY vacua in Eq. (54). They are the valleys for new smooth hybrid inflation in the case with  $\gamma \neq 0$ , but small. We should recall, however, that these two valleys are not discrete, but belong to a continuum of valleys.

Unfortunately, it is quite difficult to find a reliable expansion for the fields on these valleys, mainly because of the obstacle at  $|S| = S_c$ , which prevents us from taking the limit  $v_g/|S| \rightarrow 0$ . So, numerical computation is our last resort. We have found numerically that, when the system crosses the critical point at  $\sigma = \sigma_c$  ( $\sigma_c \equiv \sqrt{2}S_c$ ) after it has rolled down the trivial flat direction, does not immediately settle down on the new smooth path. This takes place after a while and at a value of  $\sigma$  which is well above  $0.95\sigma_c$ . Furthermore, quantum fluctuations which could kick the system out of the new smooth path are utterly suppressed well before the system reaches this value of  $\sigma$ . However, to be on the safe side, we will consider here the slow rolling of the system along the new smooth



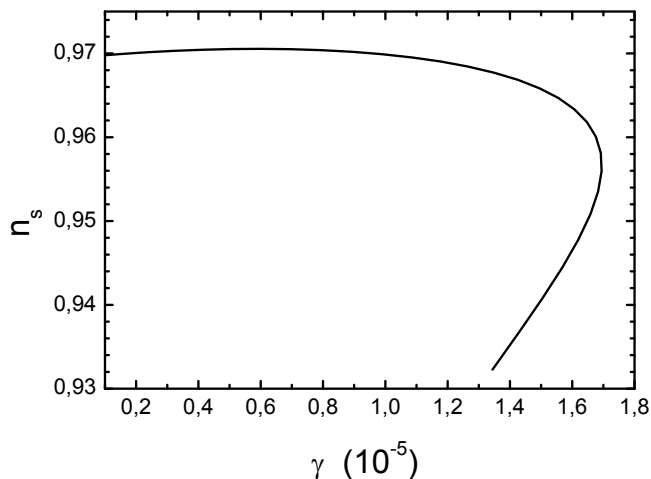


FIG. 3: Spectral index in new smooth hybrid inflation versus  $\gamma$  in global SUSY for  $p = \sqrt{2}\kappa M/m = 1/\sqrt{2}$  and  $\kappa = 0.1$ . The endpoint of the curve at  $n_s \simeq 0.932$  corresponds to the case where our present horizon scale crosses outside the inflationary horizon when  $\sigma = 0.95\sigma_c$ .

path starting from  $\sigma = 0.95\sigma_c$ . In Fig. 2, we plot the number of e-foldings  $N_{\text{nsi}}$  along the new smooth path as a function of the parameter  $\gamma$  in global SUSY and with the parameter values in Eq. (46) when the system slowly rolls from  $\sigma = 0.95\sigma_c$  down to  $\sigma = \sigma_f$ , where  $\eta = -1$  and the slow rolling ends. We see that, for small enough  $\gamma$ , we can have an adequate number of e-foldings for solving the horizon and flatness problems of standard hot big bang cosmology.

To pursue the investigation of the model further, we set  $p = \sqrt{2}\kappa M/m = 1/\sqrt{2}$ ,  $\kappa = 0.1$  and fix the value of the power spectrum  $P_{\mathcal{R}}$  of the primordial curvature perturbation to the three-year WMAP [22] result  $P_{\mathcal{R}}^{1/2} \simeq 4.85 \cdot 10^{-5}$ . As already mentioned, on the new smooth path, the fields  $H^c$  and  $\bar{H}^{c*}$  have practically the same phase ( $\theta \simeq 0$ ). So, one of the vacua in Eq. (54) is already selected during inflation (i.e. the common phase of  $H^c$  and  $\bar{H}^{c*}$  is fixed during inflation). We set the VEV  $|H^c| = \sqrt{m\phi_+/\lambda}$  equal to the SUSY GUT scale (in practice, we just put  $v_g = \sqrt{mM/\lambda} \simeq 2.86 \cdot 10^{16}$  GeV, since the resulting error is very small). After these choices, the only freedom left is the value of  $\gamma$ . In Fig. 3, we plot the predicted spectral index of the model as a function of  $\gamma$ . We terminate the curve when the value of  $\sigma$  at which our present horizon crosses outside the inflationary horizon becomes as large as  $0.95\sigma_c$ . We observe that there exists a range of values for  $\gamma$  within which the system admits two separate solutions, each corresponding to a different value of  $\lambda$ . This new feature of the model, which is not shared by conventional smooth hybrid inflation, originates from the presence of the critical point at  $\sigma = \sigma_c$  blocking the extension of the new smooth path to larger values of  $\sigma$ . The part of the curve with  $n_s < 0.96$  corresponds to values of  $\sigma_Q$  in the range

$0.85 < \sigma_Q/\sigma_c < 0.946$ , while its branch with  $n_s > 0.96$  corresponds to  $\sigma_Q/\sigma_c < 0.85$ . We see that spectral indices compatible with Eq. (45) can easily be obtained for  $\gamma$ 's which are small enough so that the number of e-foldings generated is adequately large for solving the horizon and flatness problems. It is important to point out that, in global SUSY, the new smooth hybrid inflation model is far superior to conventional smooth hybrid inflation, which predicts [12]  $n_s \simeq 0.969$ , in that it can easily accommodate much smaller values of  $n_s$  and, thus, be more comfortably compatible with data. However, we should note that obtaining values of  $n_s$  which are very close to its lower bound in Eq. (45) would require getting slightly above  $\sigma = 0.95\sigma_c$ , which is not impossible at all as we already explained.

For the values of  $\gamma$  which correspond to the curve depicted in Fig. 3, i.e.  $\gamma \simeq (0.3 - 1.7) \cdot 10^{-5}$ , we find that  $\lambda \simeq (1.4 - 3.1) \cdot 10^{-3}$ ,  $M \simeq (2.4 - 3.6) \cdot 10^{16}$  GeV,  $m \simeq (4.8 - 9.2) \cdot 10^{15}$  GeV, and  $\sigma_c \simeq (3 - 9) \cdot 10^{17}$  GeV. The number of e-foldings from the time when the pivot scale  $k_0$  crosses outside the inflationary horizon until the end of inflation is  $N_Q \simeq 53.6 - 53.85$ . The value  $\sigma_f$  of  $\sigma$  when inflation ends is about  $1.4 \cdot 10^{17}$  GeV and  $\sigma_Q$  lies in the range  $(2.85 - 3.025) \cdot 10^{17}$  GeV. Finally,  $dn_s/d\ln k \simeq -(4.1 - 5.5) \cdot 10^{-4}$  and  $r \simeq (3 - 13) \cdot 10^{-7}$ . Variations in the values of  $p$  and  $\kappa$  (which are the only arbitrarily chosen parameters) have shown not to have any significant effect on the results. Contrary to the  $\gamma = 0$  case, the numerical results for  $\gamma \neq 0$  certainly depend on the choice of the phases of the parameters in the superpotential of Eq. (1). As already explained, only one of the dimensionless parameters of this superpotential, say the parameter  $\gamma$ , is genuinely complex. Its phase affects the position of the SUSY vacua in Eq. (54) and presumably the position of the new smooth paths which lead to these vacua. However, the general qualitative structure of the theory is not expected to be affected.

### III. SUPERGRAVITY CORRECTIONS

Following Refs. [25, 26], we will show that when global SUSY is promoted to local, some features of the model are sensitive to non-minimal terms in the Kähler potential. In particular, although SUGRA corrections with a minimal Kähler potential raise the spectral index above the allowed range, non-minimal terms can help us to reduce the spectral index so as to become comfortably compatible with the data. Once again, we will first concentrate on the case  $\gamma = 0$  and then extend the model to include a small, but non-zero  $\gamma$ .

The F-term scalar potential in SUGRA is given by

$$V = e^{K/m_{\text{P}}^2} \left[ (F_i)^* K^{i^*j} F_j - 3 \frac{|W|^2}{m_{\text{P}}^2} \right], \quad (65)$$

where  $K$  is the Kähler potential,  $F_i = W_i + K_i W/m_{\text{P}}^2$ , a subscript  $i$  ( $i^*$ ) denotes derivation with respect to the

complex scalar field  $s^i$  ( $s^{i*}$ ) and  $K^{i*j}$  is the inverse of the Kähler metric  $K_{j i^*}$ .

We will consider, at first, a minimal Kähler potential and leave the inclusion of non-minimal terms for later. The minimal Kähler potential, in our case, has the form

$$K_0 = |S|^2 + |\phi|^2 + |\bar{\phi}|^2 + |H^c|^2 + |\bar{H}^c|^2 \quad (66)$$

and the scalar potential is given by

$$\tilde{V}_0 \equiv \frac{V_0}{\kappa^2 M^4} = e^{K_0/m_{\text{P}}^2} \left[ \sum_s \left| \tilde{W}_s + \frac{\tilde{W}_s^*}{m_{\text{P}}^2} \right|^2 - 3 \frac{|\tilde{W}|^2}{m_{\text{P}}^2} \right], \quad (67)$$

where  $\tilde{W} = W/\kappa M^2$  and  $s$  stands for any of the five complex scalar fields appearing in Eq. (66). We have verified numerically that, for the parameters in Eq. (46) and  $\gamma = 0$ , the potential is again minimized for fixed  $|S|$  on the new smooth path with  $\phi = \pm|\phi|$ ,  $\bar{\phi} = \pm|\bar{\phi}|$  and  $H^c \bar{H}^c = \pm|H^c|^2$ , where the signs are correlated (recall that  $S$  is chosen real and positive). So, we will restrict our attention again to these directions. Furthermore, we have found that the relative error in approximating the new smooth path by Eq. (23) or (31) is of the same order of magnitude as that in the global SUSY limit (see Fig. 1), namely  $\sim 1\%$ . So, we will use again these expansions for the new smooth path in SUGRA.

Below, we give the expansions of the various quantities entering the potential of Eq. (67) calculated on the new smooth path for  $\gamma = 0$ . Note that, besides  $w$ , we now have another small variable, namely  $|S|/m_{\text{P}}$ , which is expected to be at least one order of magnitude below unity during inflation (e.g.  $S_Q/m_{\text{P}} \sim 0.08$  for the relevant value of  $v_g$ ). In addition, the constants  $v_g/m_{\text{P}}$  and  $M/m_{\text{P}}$  are also well below unity. We will treat only  $v_g/m_{\text{P}}$  as an independent small constant since  $M/m_{\text{P}} = M/v_g \cdot v_g/m_{\text{P}}$  with  $M/v_g \sim 1$ . Using Eqs. (23) and (24)-(28), we can expand the superpotential and its derivatives on the new smooth path as follows:

$$\frac{\tilde{W}}{m_{\text{P}}} \simeq \frac{|S|}{m_{\text{P}}} \left[ 1 - \frac{1}{2} x_2 (1 - 4x_2) w^4 + \dots \right], \quad (68)$$

$$\tilde{W}_S \simeq \left[ 1 - x_2^2 w^4 + \dots \right], \quad (69)$$

$$\tilde{W}_{\phi} \simeq \pm \left[ -4x_2 z_2 \frac{M}{v_g} w^3 + \dots \right], \quad (70)$$

$$\tilde{W}_{\bar{\phi}} \simeq \pm \left[ 2\sqrt{2} p x_2^2 w^2 + \dots \right], \quad (71)$$

$$\tilde{W}_{H^c} = \pm \tilde{W}_{\bar{H}^c} \simeq \left[ -2x_2 \sqrt{z_2} w^2 + \dots \right], \quad (72)$$

where the  $\pm$  signs are again correlated, the ellipses represent terms of higher order in  $w$ , and Eq. (72) has been written in the case where  $H^c > 0$  (for  $H^c < 0$ , we should put an overall minus sign in front of the bracket). Using Eq. (23), we can write the expansions of the fields on the

new smooth path as

$$\frac{\phi}{m_{\text{P}}} \simeq \pm \frac{M}{m_{\text{P}}} \left[ x_2 w^2 + x_4 w^4 + \dots \right], \quad (73)$$

$$\frac{\bar{\phi}}{m_{\text{P}}} \simeq \pm \sqrt{2} p \frac{v_g}{m_{\text{P}}} \left[ y_1 w + y_3 w^3 + \dots \right], \quad (74)$$

$$\frac{H^c}{m_{\text{P}}} = \pm \frac{\bar{H}^c}{m_{\text{P}}} \simeq \frac{v_g}{m_{\text{P}}} \left[ \sqrt{z_2} w + \frac{z_4}{2\sqrt{z_2}} w^3 + \dots \right], \quad (75)$$

where the  $\pm$  signs are correlated with the ones in Eqs. (70)-(72) and we again take the case  $H^c > 0$ .

We will seek an expansion of the dimensionless potential  $\tilde{V}_0$  on the new smooth path (for  $\gamma = 0$ ) in powers of  $|S|/m_{\text{P}}$  and  $w$ . One can easily show, using Eqs. (67)-(75), that only even powers of  $|S|/m_{\text{P}}$  and  $w$  enter this expansion. Thus, the dimensionless potential expanded in these variables up to fourth order takes the form

$$\tilde{V}_0 \simeq A_0 + A_2 \frac{|S|^2}{m_{\text{P}}^2} + A_4 \frac{|S|^4}{m_{\text{P}}^4} + B_2 w^2 + B_4 w^4. \quad (76)$$

To construct the expansion of the dimensionless potential on the new smooth inflationary path, we first classify the various possible types of dimensionless quantities entering the calculation of  $\tilde{V}_0$  on this path. The dimensionless parameters  $p$ ,  $x_i$ ,  $y_i$ ,  $z_i$ ,  $\lambda/\kappa$ , and  $M/v_g$  will be considered to be of order unity and, as all the quantities of order unity, will be called of type 1. Any quantity that is proportional to some positive power of  $w = v_g/|S|$  with coefficient of order unity will be called of type  $t_1$ . Note that all the terms in the square brackets in Eqs. (68)-(75) are either of type 1 or  $t_1$ . Furthermore, any quantity that is proportional to some positive power of  $|S|/m_{\text{P}}$  with coefficient of order unity will be called of type  $t_2$ . Finally, positive powers of the small constant  $v_g/m_{\text{P}}$  with coefficients of order unity will be called quantities of type  $m$ . It is easy to see, using Eqs. (67)-(75), that only even powers of  $v_g/m_{\text{P}}$  appear in the expansion of  $\tilde{V}_0$ . Quantities of the form  $t_1 \cdot t_2$  can only take one of the forms  $m$ ,  $m \cdot t_1$ , and  $m \cdot t_2$ . So, the final expansion of  $\tilde{V}_0$  is expected to contain only terms of the form 1,  $t_1$ ,  $t_2$ ,  $m$ ,  $m \cdot t_1$ , and  $m \cdot t_2$ .

Now, we can split the relevant range  $v_g \lesssim |S| \lesssim m_{\text{P}}$  of  $|S|$  into two intervals according to which of the two fourth order quantities  $v_g^4/|S|^4$  and  $|S|^4/m_{\text{P}}^4$  dominates. The former dominates in the interval  $v_g \lesssim |S| \lesssim (v_g m_{\text{P}})^{1/2}$ , while the latter in the interval  $(v_g m_{\text{P}})^{1/2} \lesssim |S| \lesssim m_{\text{P}}$ . Comparing the quantity  $v_g^2/m_{\text{P}}^2$  with the two aforementioned fourth order quantities, we find that, in each of the two intervals, it is smaller than the dominant fourth order quantity in this interval. So, all the terms of type  $m$  can be neglected in the final expression of the potential in Eq. (76) provided that  $A_4$  and  $B_4$  contain terms of type 1, which turns out to be the case (see below). The same is true for the terms of order  $v_g^2/m_{\text{P}}^2 \cdot v_g^2/|S|^2$  and  $v_g^2/m_{\text{P}}^2 \cdot |S|^2/m_{\text{P}}^2$  as well as all the higher order terms of the form  $m \cdot t_1$  and  $m \cdot t_2$ . According to the above, the dimensionless potential to fourth order in  $|S|/m_{\text{P}}$  and  $w$

should only contain terms of type 1,  $t_1$ , and  $t_2$ , which is equivalent to saying that the coefficients  $A_i$  and  $B_i$  in Eq. (76) should not contain terms of type  $m$ .

Let us now find some rules which can help us manipulate the expansion of  $\tilde{V}_0$  on the new smooth path. First of all, note that this dimensionless potential consists of a sum of products of  $\tilde{W}/m_P$ ,  $\tilde{W}_s$ , and  $|s|/m_P$ , as seen from Eq. (67). The quantities  $|s|/m_P$  with  $s \neq S$  in Eqs. (73)-(75) consist of terms of the form  $m \cdot t_1$ , while  $|S|/m_P$  and the quantities in Eqs. (68)-(72) contain terms of the form 1,  $t_1$ ,  $t_2$ , and  $m \cdot t_1$ . It is readily shown that products of any of these quantities can only contain terms of type 1,  $t_1$ ,  $t_2$ ,  $m$ ,  $m \cdot t_1$ , and  $m \cdot t_2$ . Moreover, one can easily see that, if a term of type  $m$ ,  $m \cdot t_1$ , or  $m \cdot t_2$  is encountered at any intermediate stage of the calculation, it is bound to yield terms of type  $m$ ,  $m \cdot t_1$ , or  $m \cdot t_2$  in the final expansion of  $\tilde{V}_0$ . However, we have already shown that such terms should not be kept in the final form of the potential since they give a negligible contribution. Thus, we conclude that we can drop terms of the form  $m$ ,  $m \cdot t_1$ , and  $m \cdot t_2$  whenever we come across them and maintain only terms of the form 1,  $t_1$ , and  $t_2$  in the various stages of the calculation. A corollary to this is that we can take  $K_0$  in the exponential of Eq. (67) to be simply  $|S|^2$  and  $\tilde{W}/m_P$  in Eq. (68) to be simply  $|S|/m_P$ .

Taking all the above into account, we can now quite easily find that the relevant terms in the dimensionless potential of Eq. (68) on the new smooth path (for  $\gamma = 0$ ) will be all contained in

$$\tilde{V}_0 \simeq e^{|S|^2/m_P^2} \left[ \tilde{V}_g + \frac{|\tilde{W}|^2 |S|^2}{m_P^4} + \left( \frac{\tilde{W}_S^* \tilde{W}_S}{m_P^2} + \text{c.c.} \right) - 3 \frac{|\tilde{W}|^2}{m_P^2} \right], \quad (77)$$

where  $\tilde{V}_g = \sum_s |\tilde{W}_s|^2$  is the dimensionless scalar potential in the global SUSY limit. Substituting Eqs. (68)-(72) into Eq. (77) and keeping only the relevant terms, we obtain the potential

$$V_0 \simeq \kappa^2 M^4 \left( 1 + \frac{1}{2} \frac{|S|^4}{m_P^4} - \frac{v_g^4}{54|S|^4} \right). \quad (78)$$

Note that, in our case, the leading SUGRA correction to the inflationary potential for minimal Kähler potential, which corresponds to the second term in the parenthesis in the right hand side of Eq. (78), is the same as the one found in Ref. [4] in the case of standard hybrid inflation and in Ref. [23] in the case of shifted and smooth hybrid inflation. Actually, the inflationary potential for conventional smooth hybrid inflation in Ref. [23] coincides with the potential in Eq. (78), which applies to new smooth hybrid inflation for  $\gamma = 0$ .

Let us now turn to the consideration of a more general Kähler potential containing non-minimal terms. As we are interested in the region of field space with  $|s| \ll m_P$ , we can expand the Kähler potential as a power series in

the fields. The same rules that we have extracted above for manipulating the expansion of the potential on the new smooth path in the case of minimal Kähler potential hold for this case as well. In particular, in expanding the potential up to fourth order in  $|S|/m_P$  and  $w$ , we can drop terms of the form  $m$ ,  $m \cdot t_1$ , and  $m \cdot t_2$  whenever they appear at an intermediate stage of the calculation. As a consequence, we can take  $K$  in the exponential of Eq. (65) to consist only of terms containing solely powers of the field  $S$  and not the other fields (compare with the similar argument above in the case of a minimal Kähler potential). Since terms of the form  $|S|^n (S^m + S^{*m})$  with  $n \geq 0$  and  $m \geq 1$  are not allowed due to the R symmetry, the only relevant non-minimal Kähler potential terms are

$$|S|^4/m_P^2, \quad |S|^6/m_P^4 \quad (79)$$

up to order six in  $|S|/m_P$ . The same terms are the only non-minimal Kähler potential terms (up to sixth order) which can give a non-negligible contribution to  $K_i/m_P$ . This is due to the fact that, in  $K$ , we cannot have terms with a single field  $s \neq S$  multiplying powers of  $S$  and  $S^*$  since there exist no other gauge singlet fields in the theory. So, all terms in  $K$  other than the ones of the form in Eq. (79) contain at least two fields  $s \neq S$  and, thus, give negligible contributions to  $K_i/m_P$ . Finally, the inverse Kähler metric  $K^{i^*j}$  can be expanded as a power series of the higher order terms contained in the Kähler metric  $K_{j i^*}$ . Besides the terms of the form in Eq. (79), other Kähler potential terms that can contribute to  $K_{j i^*}$  are certainly the ones of the form

$$|S|^2 |s|^2/m_P^2 \quad (80)$$

with  $s$  being any of the fields  $\phi$ ,  $\bar{\phi}$ ,  $H^c$ , and  $\bar{H}^c$ . In general, any terms containing two of the four fields  $\phi$ ,  $\bar{\phi}$ ,  $H^c$ , and  $\bar{H}^c$  multiplied by powers of  $S$  and  $S^*$  will contribute. The only possible combinations of two fields  $s \neq S$  other than  $|s|^2$  that respect gauge invariance are  $H^c \bar{H}^c$ ,  $\phi^2$ ,  $\phi \bar{\phi}$ ,  $\phi^* \bar{\phi}$ , and  $\bar{\phi}^2$  along with their complex conjugates. The first two can be multiplied by powers of  $|S|^2$ , while the other three need some extra  $S$  or  $S^*$  factors in order to become R symmetry invariant. In summary, we can parameterize the most general Kähler potential which is relevant for our calculation here as follows:

$$K = K_0 + \frac{k_S}{4} \frac{|S|^4}{m_P^2} + \frac{k_{SS}}{6} \frac{|S|^6}{m_P^4} + \sum_{s \neq S} k_{Ss} \frac{|S|^2 |s|^2}{m_P^2} + \left( k_{\phi \bar{\phi} S^*} \frac{\phi \bar{\phi} S^*}{m_P} + k_{\phi^* \bar{\phi} S^*} \frac{\phi^* \bar{\phi} S^*}{m_P} + k_{\bar{\phi} \bar{\phi} S^* S^*} \frac{\bar{\phi}^2 S^{*2}}{m_P^2} + k_{\phi \phi S S^*} \frac{\phi^2 |S|^2}{m_P^2} + k_{H \bar{H} S S^*} \frac{H^c \bar{H}^c |S|^2}{m_P^2} + \text{c.c.} \right), \quad (81)$$

where the various  $k$  coefficients are considered to be of order unity. From this, we get

$$\frac{K_S}{m_P} \simeq \frac{S^*}{m_P} \left( 1 + \frac{k_S}{2} \frac{|S|^2}{m_P^2} + \frac{k_{SS}}{2} \frac{|S|^4}{m_P^4} \right), \quad (82)$$

while all the other first derivatives  $K_s/m_P$  are of the form  $m$ ,  $m \cdot t_1$ , or  $m \cdot t_2$  and can be neglected. The relevant contributions to the Kähler metric and its inverse are

$$(K_{ji^*}) \simeq \begin{pmatrix} K_{11^*} & 0 & 0 & 0 & 0 \\ 0 & K_{22^*} & K_{23^*} & 0 & 0 \\ 0 & K_{32^*} & K_{33^*} & 0 & 0 \\ 0 & 0 & 0 & K_{44^*} & 0 \\ 0 & 0 & 0 & 0 & K_{55^*} \end{pmatrix}, \quad (83)$$

$$(K^{i^*j}) \simeq \begin{pmatrix} \frac{1}{K_{11^*}} & 0 & 0 & 0 & 0 \\ 0 & \frac{K_{33^*}}{D} & -\frac{K_{23^*}}{D} & 0 & 0 \\ 0 & -\frac{K_{32^*}}{D} & \frac{K_{22^*}}{D} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{K_{44^*}} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{K_{55^*}} \end{pmatrix}, \quad (84)$$

where

$$K_{11^*} \simeq 1 + k_S \frac{|S|^2}{m_P^2} + \frac{3}{2} k_{SS} \frac{|S|^4}{m_P^4}, \quad (85)$$

$$K_{22^*} = 1 + k_{S\phi} \frac{|S|^2}{m_P^2}, \quad K_{33^*} = 1 + k_{S\bar{\phi}} \frac{|S|^2}{m_P^2}, \quad (86)$$

$$K_{44^*} = 1 + k_{SH} \frac{|S|^2}{m_P^2}, \quad K_{55^*} = 1 + k_{S\bar{H}} \frac{|S|^2}{m_P^2}, \quad (87)$$

$$K_{23^*} = K_{32^*}^* = k_{\phi^* \bar{\phi} S^*}^* \frac{S}{m_P}, \quad (88)$$

$$D = K_{22^*} K_{33^*} - |K_{23^*}|^2, \quad (89)$$

and  $i = 1, 2, 3, 4, 5$  correspond to the fields  $S$ ,  $\phi$ ,  $\bar{\phi}$ ,  $H^c$ ,  $\bar{H}^c$  respectively.

As can be seen from Eqs. (65) and (84), the only contribution to the scalar potential on the new smooth path originating from non-diagonal elements of the inverse Kähler metric comes from the term  $(F_2)^* K^{2^*3} F_3 + \text{c.c.}$ , which, on the new smooth path, can be approximated to leading order by

$$\kappa^2 M^4 \left( 16\sqrt{2} p x_2^3 z_2 \text{Re } k_{\phi^* \bar{\phi} S^*} \right) \frac{M}{m_P} w^4. \quad (90)$$

It is, thus, of the form  $m \cdot t_1$  and can be dropped. From the diagonal entries in the inverse Kähler metric, one finds that the relevant contributions to the potential on the new smooth path will come from

$$V \simeq e^{K/m_P^2} \left[ \left| W_S + \frac{W K_S}{m_P^2} \right|^2 K^{S^*S} + \sum_{s \neq S} |W_s|^2 - 3 \frac{|W|^2}{m_P^2} \right]. \quad (91)$$

Substituting Eqs. (68)-(72), (82), (84), and (85) into Eq. (91), expanding in powers of  $|S|/m_P$ , and keeping only terms of type  $1$ ,  $t_1$  and  $t_2$ , we finally obtain, for the

potential on the new smooth path for  $\gamma = 0$  in SUGRA, the approximation

$$V \simeq v_0^4 \left( 1 - k_S \frac{|S|^2}{m_P^2} + \frac{1}{2} \gamma_S \frac{|S|^4}{m_P^4} - \frac{v_g^4}{54|S|^4} \right), \quad (92)$$

where  $v_0 = \sqrt{k}M$  and  $\gamma_S \equiv 1 - \frac{7}{2} k_S - 3 k_{SS} + 2 k_S^2$ . We see that, from the variety of terms in the Kähler potential, only those with coefficients  $k_S$  and  $k_{SS}$  contribute to the scalar potential on the new smooth path expanded up to fourth order in  $|S|/m_P$  and  $v_g/|S|$ . Note that Eq. (92) coincides with the corresponding result found in Ref. [26] in the case of conventional smooth hybrid inflation. Moreover, the SUGRA correction to the inflationary potential which corresponds to the second and third terms in the parenthesis in the right hand side of Eq. (92) coincides with the SUGRA correction found in Ref. [25] in the case of standard hybrid inflation.

All the above results hold as long as Eq. (31) is a good approximation to the new smooth path for  $\gamma = 0$  in the case of a non-minimal Kähler potential too. We have checked numerically that, at least for values of the parameters close to the ones in Eq. (46), the relative error in the fields on the new smooth path remains smaller than 2% for a general Kähler potential (which can include more terms besides the ones shown in Eq. (81)) even when the various  $k$  coefficients are of order unity.

As in Ref. [25], the new terms in the inflationary potential which originate from the non-minimal terms in the Kähler potential and are proportional to  $|S|^2$  and  $|S|^4$  can give rise to a local minimum at  $|S| = |S|_{\min}$  and maximum at  $|S| = |S|_{\max} < |S|_{\min}$  of the potential on the inflationary path. This means that, if the system starts from a point with  $|S| > |S|_{\max}$ , it can be trapped in the local minimum of the potential. Nevertheless, as in Ref. [26] where conventional smooth hybrid inflation was considered, in the case of new smooth hybrid inflation too, there exists a range of values for  $k_S$  where the minimum-maximum of the inflationary potential does not appear and the system can start its slow rolling from any point on the inflationary path without the danger of getting trapped.

Let us find the condition for the inflationary potential in Eq. (92), which holds in the case  $\gamma = 0$ , not to have the “minimum-maximum” problem. Using the dimensionless real inflaton field  $\hat{\sigma} \equiv \sigma/m_P$ , this potential and its derivative with respect to  $\hat{\sigma}$  are given by

$$\tilde{V} \equiv \frac{V}{v_0^4} \simeq 1 - \frac{1}{2} k_S \hat{\sigma}^2 + \frac{1}{8} \gamma_S \hat{\sigma}^4 - \frac{2\hat{v}_g^4}{27\hat{\sigma}^4}, \quad (93)$$

$$\frac{d\tilde{V}}{d\hat{\sigma}} \equiv \frac{1}{v_0^4} \frac{dV}{d\hat{\sigma}} \simeq -k_S \hat{\sigma} + \frac{1}{2} \gamma_S \hat{\sigma}^3 + \frac{8\hat{v}_g^4}{27\hat{\sigma}^5}, \quad (94)$$

where  $\hat{v}_g \equiv v_g/m_P$  and  $\gamma_S$  is assumed positive. We can evade the local maximum and minimum of the inflationary potential if we require that  $d\tilde{V}/d\hat{\sigma}$  remains positive for any  $\hat{\sigma} > 0$  so that this potential is a monotonically

increasing function of  $\sigma$ . This gives the condition

$$f(\hat{\sigma}) \equiv \hat{\sigma}^8 - \frac{2k_S}{\gamma_S} \hat{\sigma}^6 + \frac{16\hat{v}_g^4}{27\gamma_S} \gtrsim 0. \quad (95)$$

For  $k_S > 0$ , which is the interesting case as we will soon see, the minimum of  $f(\hat{\sigma})$  lies at  $\hat{\sigma}_1 = (3k_S/2\gamma_S)^{1/2}$ , where  $f(\hat{\sigma}_1) = -27k_S^4/16\gamma_S^4 + 16\hat{v}_g^4/27\gamma_S$  and the requirement that  $f(\hat{\sigma}_1) \gtrsim 0$  yields the restriction

$$k_S \lesssim k_S^{\max} \equiv \frac{4}{3\sqrt{3}} \gamma_S^{3/4} \frac{v_g}{m_P}. \quad (96)$$

Note that, for  $\gamma_S \sim 1$ , this inequality implies that  $\hat{\sigma}_1 < 1$  and, thus, the minimum of  $f(\hat{\sigma})$  lies in the relevant region where  $\sigma < m_P$ .

For  $k_S \gtrsim k_S^{\max}$ , on the other hand, the inflationary potential has a local minimum and maximum which approximately lie at

$$\sigma_{\min} \simeq m_P \left( \frac{2k_S}{\gamma_S} \right)^{1/2}, \quad \sigma_{\max} \simeq m_P \left( \frac{8v_g^4}{27k_S m_P^4} \right)^{1/6}. \quad (97)$$

Even in this case, the system can always undergo hybrid inflation with the required number of e-foldings starting at a  $\sigma < \sigma_{\max}$ . This is due to the vanishing of the derivative  $V^{(1)}$  at  $\sigma = \sigma_{\max}$ . However, the more the e-foldings we want to obtain the closer we must set the initial  $\sigma$  to the maximum of the potential, which leads to an initial condition problem. Yet, as we will see, we can obtain a spectral index as low as 0.95 at  $k_0 = 0.002 \text{ Mpc}^{-1}$  in agreement with the WMAP three-year value  $0.958 \pm 0.016$  [22] maintaining the constraint  $k_S \lesssim k_S^{\max}$ .

Using the inflationary potential in Eq. (92), the spectral index of density perturbations turns out to be

$$n_s \simeq 1 + 2\eta_Q \simeq 1 - 2k_S + 3\gamma_S \frac{\sigma_Q^2}{m_P^2} - \frac{80v_g^4 m_P^2}{27\sigma_Q^6}, \quad (98)$$

where  $\eta_Q$  is the value of  $\eta$  when the pivot scale  $k_0 = 0.002 \text{ Mpc}^{-1}$  crosses outside the inflationary horizon. We can see that the  $k_S$  term in the Kähler potential contributes to the lowering of the spectral index if  $k_S$  is positive. So, a  $k_S$  with this choice of its sign can help us to make the spectral index comfortably compatible with the three-year WMAP measurements [22]. However, since we cannot have any reliable and convenient approximation for  $\sigma_Q$ , a numerical investigation is required.

Turning now to the case of small, but non-zero  $\gamma$ , one can assert that again only the same non-minimal terms of the Kähler potential with coefficients  $k_S$  and  $k_{SS}$  will enter the expansion of the potential on the new smooth path, although the global SUSY potential for new smooth hybrid inflation is not, in this case, given by Eq. (33) but has to be calculated numerically. So, due to the small value of  $\gamma$ , we can assume that the potential on the new smooth path in the case of SUGRA with the non-minimal

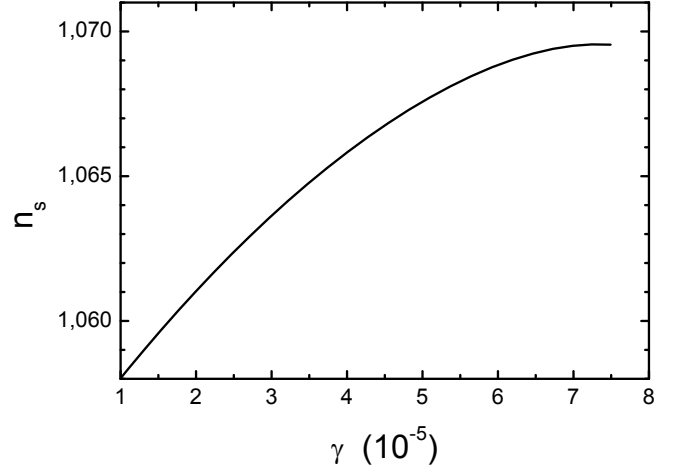


FIG. 4: Spectral index in new smooth hybrid inflation versus  $\gamma$  in minimal SUGRA for  $p = \sqrt{2}\kappa M/m = 1/\sqrt{2}$ ,  $\kappa = 0.1$ . The endpoint of the curve at  $\gamma \simeq 0.75 \cdot 10^{-6}$  ( $n_s \simeq 1.0695$ ) corresponds to the case where our present horizon scale crosses outside the inflationary horizon when  $\sigma = 0.95 \sigma_c$ .

Kähler potential of Eq. (81) and  $\gamma \neq 0$  has the form

$$V \simeq v_0^4 \left( \tilde{V}_{\text{SUSY}} - \frac{1}{2} k_S \frac{\sigma^2}{m_P^2} + \frac{1}{8} \gamma_S \frac{\sigma^4}{m_P^4} \right), \quad (99)$$

where  $\tilde{V}_{\text{SUSY}} \equiv V_{\text{SUSY}}/v_0^4$  with  $V_{\text{SUSY}}$  being the inflationary potential in the case of global SUSY and  $\gamma \neq 0$ . Note, also, that, in the SUGRA and  $\gamma \neq 0$  case, the critical value of  $\sigma$ , where the trivial flat direction becomes unstable, will be slightly different from the critical value of  $\sigma$  in the global SUSY case.

As in the global SUSY case with  $\gamma \neq 0$ , we take  $p = \sqrt{2}\kappa M/m = 1/\sqrt{2}$ ,  $\kappa = 0.1$  and fix numerically the power spectrum  $P_{\mathcal{R}}$  of the primordial curvature perturbation to the three-year WMAP normalization [22] in SUGRA too. We also set the VEV  $|\langle H^c \rangle|$  equal to the SUSY GUT scale, which, to a very good approximation, means that we put  $v_g = \sqrt{mM/\lambda} \simeq 2.86 \cdot 10^{16} \text{ GeV}$ . The scalar spectral index in SUGRA with a minimal Kähler potential (i.e.  $k_S = k_{SS} = 0$ ) as a function of the parameter  $\gamma$  is shown in Fig. 4. We terminate the curve when the value of  $\sigma$  at which our present horizon scale crosses outside the inflationary horizon reaches  $0.95 \sigma_c$ . We see that minimal SUGRA elevates the scalar spectral index above the 95% confidence level range obtained by fitting the three-year WMAP data [22] by the standard power-law  $\Lambda\text{CDM}$  cosmological model ( $n_s$  tends to approximately 1.055 as  $\gamma \rightarrow 0$ ). This situation is readily rectified by the inclusion of non-minimal terms in the Kähler potential as we will see below. For the range of values of  $\gamma$  shown in Fig. 4 (i.e. for  $\gamma \sim (1 - 7.5) \cdot 10^{-5}$ ), the ranges of the other parameters of the model are as follows:  $\lambda \simeq (1.33 - 1.68) \cdot 10^{-2}$ ,  $M \simeq (7.4 - 8.3) \cdot 10^{16} \text{ GeV}$ ,  $m \simeq (1.48 - 1.66) \cdot 10^{16} \text{ GeV}$ ,  $\sigma_c \simeq (4.2 - 9.8) \cdot 10^{17} \text{ GeV}$ ,  $\sigma_Q \simeq (3.6 - 3.95) \cdot 10^{17} \text{ GeV}$ ,

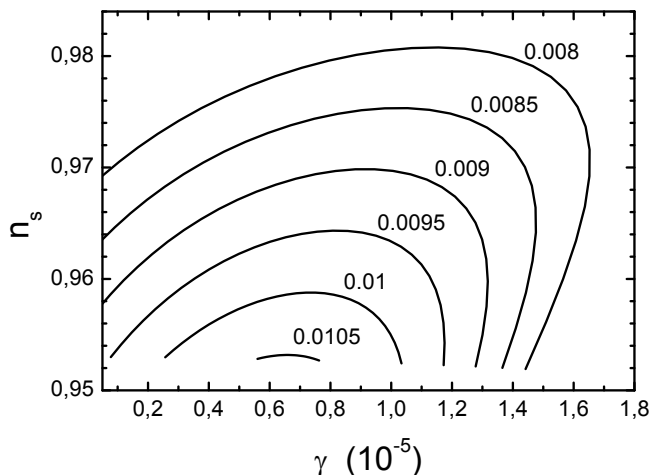


FIG. 5: Spectral index in new smooth hybrid inflation in non-minimal SUGRA as a function of  $\gamma$  for  $p = \sqrt{2}\kappa M/m = 1/\sqrt{2}$  and  $\kappa = 0.1$ . The values of  $k_S$ , which are indicated on the curves, range from 0.008 to 0.0105 and  $k_{SS} = 0$ .

$\sigma_f \simeq (1.39 - 1.395) \cdot 10^{17} \text{ GeV}$ ,  $N_Q \simeq 54.3 - 54.4$ ,  $dn_s/d\ln k \simeq -(2.1 - 2.6) \cdot 10^{-3}$ , and  $r \simeq (2.4 - 3.8) \cdot 10^{-5}$ .

Next, we consider the case where non-minimal terms are present in the Kähler potential. We will let  $k_S$  have a non-zero positive value, but take  $k_{SS} = 0$  for simplicity. We calculate numerically the spectral index and plot it in Fig. 5 as a function of the parameter  $\gamma$  for various values of  $k_S$ . The limiting points on each curve correspond to the situation where the potential on the new smooth inflationary path starts developing a local minimum and maximum. We observe that, although all curves terminate on the right, only curves that correspond to larger values of  $k_S$  (and smaller values of  $n_s$ ) have an endpoint on the small  $\gamma$  side. It is instructive to note that, for  $\gamma = 0$ , Eq. (96) gives  $k_S^{\text{max}} \simeq 0.0088$ , which is in fairly good agreement with Fig. 5. From this figure, one can infer that the spectral index can be readily set below unity in SUGRA with non-minimal Kähler potential and that one can achieve a value as low as  $n_s \simeq 0.952$  without having to put up with a local minimum and maximum of the potential on the inflationary path. This minimal value of  $n_s$  corresponds to the endpoint of the curve with  $k_S = 0.008$ . The maximal allowed value of  $k_S$  is about 0.01054 corresponding to  $\gamma \simeq 0.66 \cdot 10^{-5}$  and  $n_s \simeq 0.953$ . Finally, for the range of values of  $\gamma$  and  $k_S$  corresponding to the curves in Fig. 5, the ranges of variance of the other parameters of the model are as follows:  $\lambda \simeq (1.5 - 2.6) \cdot 10^{-3}$ ,  $M \simeq (2.5 - 3.3) \cdot 10^{16} \text{ GeV}$ ,  $m \simeq (0.5 - 0.66) \cdot 10^{16} \text{ GeV}$ ,  $\sigma_c \simeq (0.45 - 1.7) \cdot 10^{18} \text{ GeV}$ ,  $\sigma_Q \simeq (2.54 - 2.77) \cdot 10^{17} \text{ GeV}$ ,  $\sigma_f \simeq (1.39 - 1.395) \cdot 10^{17} \text{ GeV}$ ,  $N_Q \simeq 53.6 - 53.8$ ,  $dn_s/d\ln k \simeq -(7.2 - 9.2) \cdot 10^{-4}$ , and  $r \simeq (3 - 9.6) \cdot 10^{-7}$ . Variations in the values of  $p$  and  $\kappa$  have shown not to have any significant effect on the results. In particular, the spectral index cannot become smaller than about 0.95 by

varying these parameters provided that the appearance of a local minimum and maximum of the inflationary potential is avoided. Note, however, that smaller values of  $n_s$  can be readily achieved, but at the cost of having the minimum-maximum problem.

#### IV. CONCLUSIONS

We considered the extension of the SUSY PS model which has been introduced in Ref. [17] in order to solve the  $b$ -quark mass problem in SUSY GUT models with exact asymptotic Yukawa unification, such as the simplest SUSY PS model, and universal boundary conditions. This extended model leads naturally to a (moderate) violation of the asymptotic Yukawa unification so that, for  $\mu > 0$ , the predicted  $b$ -quark mass resides within the experimentally allowed range. Moreover, it is known that this model automatically leads to the so-called new shifted hybrid inflationary scenario, which is based only on renormalizable superpotential terms and avoids the cosmological disaster from a possible overproduction of PS magnetic monopoles at the end of inflation.

Here, we have demonstrated that this PS model can also lead to a new version of smooth hybrid inflation, which, in contrast to the conventional realization of smooth hybrid inflation, is based only on renormalizable interactions. An important prerequisite for this is that a particular parameter of the superpotential is adequately small. Then the scalar potential of the model possesses, for a wide range of its other parameters, valleys of minima with classical inclination which can be used as inflationary paths leading to a new realization of smooth hybrid inflation. This scenario, in global SUSY, is naturally consistent with the fitting of the three-year WMAP data by the standard power-law  $\Lambda$ CDM cosmological model. In particular, the spectral index turns out to be adequately small so that it is compatible with the data. Moreover, as in the conventional realization of smooth hybrid inflation, the PS gauge group is already broken to the SM gauge group during new smooth hybrid inflation and, thus, no topological defects are formed at the end of inflation. Therefore, the problem of possible overproduction of PS magnetic monopoles is solved.

Embedding the model in SUGRA with a minimal Kähler potential raises the scalar spectral index to values which are too high to be compatible with the recent data. However, inclusion of a non-minimal term in the Kähler potential with appropriately chosen sign can help to reduce the spectral index so that it resides comfortably within the allowed range. The potential along the new smooth inflationary path, however, can remain everywhere a monotonically increasing function of the inflaton field. So, unnatural restrictions on the initial conditions for inflation due to the appearance of a maximum and a minimum of the potential on the new smooth inflationary path when such a non-minimal Kähler potential is used can be avoided.

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- [1] A.H. Guth, Phys. Rev. D **23**, 347 (1981).
  - [2] G. Lazarides, Lect. Notes Phys. **592**, 351 (2002), hep-ph/0111328; J. Phys. Conf. Ser. **53**, 528 (2006), hep-ph/0607032.
  - [3] A.D. Linde, Phys. Rev. D **49**, 748 (1994).
  - [4] E.J. Copeland, A.R. Liddle, D.H. Lyth, E.D. Stewart, and D. Wands, Phys. Rev. D **49**, 6410 (1994).
  - [5] G.R. Dvali, Q. Shafi, and R.K. Schaefer, Phys. Rev. Lett. **73**, 1886 (1994); G. Lazarides, R.K. Schaefer, and Q. Shafi, Phys. Rev. D **56**, 1324 (1997).
  - [6] J.C. Pati and A. Salam, Phys. Rev. D **10**, 275 (1974).
  - [7] G. Lazarides and C. Panagiotakopoulos, Phys. Lett. B **337**, 90 (1994); S. Khalil, G. Lazarides, and C. Pallis, *ibid.* **508**, 327 (2001).
  - [8] B. Ananthanarayan, G. Lazarides, and Q. Shafi, Phys. Rev. D **44**, 1613 (1991); Phys. Lett. B **300**, 245 (1993).
  - [9] G. Shiu and S.-H.H. Tye, Phys. Rev. D **58**, 106007 (1998).
  - [10] L.L. Everett, G.L. Kane, S.F. King, S. Rigolin, and L.-T. Wang, Phys. Lett. B **531**, 263 (2002).
  - [11] I. Antoniadis and G.K. Leontaris, Phys. Lett. B **216**, 333 (1989); I. Antoniadis, G.K. Leontaris, and J. Rizos, *ibid.* **245**, 161 (1990).
  - [12] G. Lazarides and C. Panagiotakopoulos, Phys. Rev. D **52**, R559 (1995); G. Lazarides, C. Panagiotakopoulos, and N.D. Vlachos, *ibid.* **54**, 1369 (1996); R. Jeannerot, S. Khalil, and G. Lazarides, Phys. Lett. B **506**, 344 (2001).
  - [13] G. Lazarides, M. Magg, and Q. Shafi, Phys. Lett. B **97**, 87 (1980).
  - [14] R. Jeannerot, S. Khalil, G. Lazarides, and Q. Shafi, J. High Energy Phys. **10**, 012 (2000).
  - [15] G. Lazarides, hep-ph/0011130; R. Jeannerot, S. Khalil, and G. Lazarides, hep-ph/0106035.
  - [16] R. Jeannerot, S. Khalil, and G. Lazarides, J. High Energy Phys. **07**, 069 (2002).
  - [17] M.E. Gomez, G. Lazarides, and C. Pallis, Nucl. Phys. **B638**, 165 (2002).
  - [18] G. Lazarides and C. Pallis, hep-ph/0404266; hep-ph/0406081.
  - [19] R. Hempfling, Phys. Rev. D **49**, 6168 (1994); L.J. Hall, R. Rattazzi, and U. Sarid, *ibid.* **50**, 7048 (1994).
  - [20] G. Lazarides, Q. Shafi, and C. Wetterich, Nucl. Phys. **B181**, 287 (1981); G. Lazarides and Q. Shafi, *ibid.* **B350**, 179 (1991).
  - [21] M.E. Gomez, G. Lazarides, and C. Pallis, Phys. Rev. D **67**, 097701 (2003).
  - [22] D.N. Spergel *et al.*, Astrophys. J. Suppl. Ser. **170**, 377 (2007).
  - [23] V.N. Şenoğuz and Q. Shafi, Phys. Lett. B **567**, 79 (2003).
  - [24] O. Seto and J. Yokoyama, Phys. Rev. D **73**, 023508 (2006).
  - [25] M. Bastero-Gil, S.F. King, and Q. Shafi, Phys. Lett. B **651**, 345 (2007).
  - [26] M. ur Rehman, V.N. Şenoğuz, and Q. Shafi, Phys. Rev. D **75**, 043522 (2007).
  - [27] L. Boubekeur and D. Lyth, J. Cosmol. Astropart. Phys. **07**, 010 (2005).
  - [28] B. Garbrecht, C. Pallis, and A. Pilaftsis, J. High Energy Phys. **12**, 038 (2006).
  - [29] V.N. Şenoğuz and Q. Shafi, Phys. Rev. D **71**, 043514 (2005); hep-ph/0512170.
  - [30] B.A. Bassett, S. Tsujikawa, and D. Wands, Rev. Mod. Phys. **78**, 537 (2006).
  - [31] M.Yu. Khlopov and A.D. Linde, Phys. Lett. B **138**, 265 (1984); J. Ellis, J.E. Kim, and D. Nanopoulos, *ibid.* **145**, 181 (1984); J.R. Ellis, D.V. Nanopoulos, and S. Sarkar, Nucl. Phys. **B259**, 175 (1985).